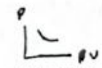
Matthew Tran
Physics 7B
MT1

- Dimensional Analysis - know it
- Dot Product $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$
- Cross Product $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$
- Taylor Series (only need 1st two terms) (a is some #)
  - $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^3(a)}{3!}(x-a)^3 + \dots$
- Integrals
  - 2D - trig sub, u-sub, by parts $\int u\,dv = uv - \int v\,du$
  - 3D - Line Integral $\int dS$
    - Surface Integral $\iint dA$
    - spherical coordinates $\rho^2 \sin\phi$ $-\pi/2 \le \phi \le \pi/2$
    - cartesian/cylindrical coordinates
- $W = \int F\,dr$
- Thermodynamic Quantities
  - Temperature (T)
  - Pressure (P)
  - Volume (V)
  - Heat (Q)
- Thermodynamic Equilibrium (all vars) - all vars stop changing
- Thermal Expansion - Anisotropic: Diff coeff in diff direction
  - Linear - $L = L_0(1+\alpha\Delta T)$
  - Area - $A = A_0(1+\gamma\Delta T)$ $\gamma = 2\alpha$
  - Volume - $V = V_0(1+\beta\Delta T)$ $\beta = 3\alpha$ } Taylor Expansion
- $PV = nRT$
  - Boyle - $PV = k$
  - Charles - $\frac{V}{T} = k$
  - Gay Lussac - $\frac{P}{T} = k$
  - Avogadro's Number - $N_A = 6.022\cdot10^{23}$
    - equal volume has same # molecules
  - Boltzmann's Constant - $K_B = 1.38\cdot10^{-23}\ \frac{J}{K}$
    - $PV = NK_BT$, N = # molecules
  - Universal Gas Constant - $R = 8.314\ \frac{J}{mol\cdot K}$
    - $R = N_AK_B$
  - Absolute Zero 0K, $P=0, T=0$
- Microscopic Theory of Gas
  - Ideal Gas
    1) Tiny particles
    2) Elastic collisions
    3) Mass, obey Newton eqn.
    4) average distance >> diameter

- Gas
  - $\bar{K} = \frac{3}{2}K_BT$; 3 DoF, per molecule
  - $E_{DoF} = \frac{1}{2}K_BT$
  - $V_{rms} = \sqrt{\frac{3K_BT}{m}}$; $m = \frac{kg}{molecule}$;
  - Maxwell Distribution
    - $f(v) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{1}{2}\frac{mv^2}{kT}}$
      - $\int_0^\infty f(v)\,dv = N$; # molecules at velocity v
    - $v_p = \sqrt{\frac{2kT}{m}}$; $\frac{d}{dv}f(v) = 0$; most probable
    - $\bar{v} = \sqrt{\frac{8}{\pi}\frac{kT}{m}}$; $\int_0^\infty v f(v)\,dv$; mean
    - $V_{rms} = \sqrt{\frac{3kT}{m}}$; $\sqrt{\int_0^\infty v^2 f(v)\,dv}$; rms
      - median kinetic energy
  - $PV = \frac{1}{3}Nm\overline{v^2}$; in 3D, approx using pressure
  - Real Gas
    - Finite size particles ; attraction; collide w/ each other
    - Van der Waal's Equation
      $\left(P + a\left(\frac{n}{V}\right)^2\right)(V - nb) = nRT$
      for attraction, finite volume
  - Equipartition Theory - $\langle E\rangle = d/2\,NkT$
    - monatomic - $\langle E\rangle = \frac{1}{2}m\langle v_x^2\rangle + \frac{1}{2}m\langle v_y^2\rangle + \frac{1}{2}m\langle v_z^2\rangle = \frac{3}{2}NkT$
    - diatomic - $\langle E\rangle =$ " $+ \frac{1}{2}I\langle\omega_x\rangle^2 + \frac{1}{2}I\langle\omega_y\rangle^2 = \frac{5}{2}NkT$
    - $V_{rms} = \sqrt{\frac{3kT}{m}}$ still, bc only dependent on translational (x, y, z)
  - Relative Humidity = $\frac{\text{partial pressure}}{\text{saturated vapor pressure}}$

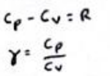
- Mean Free Path
  - $l_m = \frac{1}{4\pi\sqrt{2}r^2(N/V)}$
  - $\sqrt{2}$ correction bc depends on relative velocity
- 1st Law of Thermodynamics
  - $\Delta E_{int} = Q - W$
    - Q - heat added
    - W - work done by gas
  - $Q = mc\Delta T$; $Q = mL$
  - state variable - depends only on initial and final state
    - ex. ΔE_{int}, P, V
    - not Q, W
  - $W = \int P\,dV$



PV diagrams

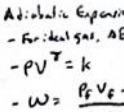
  - Heat Transfer
    - conduction - $\frac{dQ}{dt} = KA\cdot\frac{dT}{dx}$
    - convection
    - radiation $\frac{dQ}{dt} = k\epsilon AT^4$
  - 1 cal = 4.186 J ; 1 kcal = 4186 J
  - Isothermal ($\Delta T = 0$)
    - $PV = k$, $\Delta E_{int} = 0$ $W = nRT\ln\left(\frac{V_f}{V_i}\right)$
    - $Q - W = 0$; $Q = W$
  - Isobaric ($\Delta P = 0$)
    - $\frac{V}{T} = k$, $W = P\Delta V = nR\Delta T$, $Q = \Delta E_{int} + P\Delta V$ $Q = nC_p\Delta T$, $\Delta E_{int} = nC_v\Delta T$ } $C_p - C_v = R$
  - Isovolumetric ($\Delta V = 0$)
    - $\frac{P}{T} = k$, $W = 0$, $\Delta E_{int} = Q = nC_v\Delta T$
  - Adiabatic ($Q = 0$)
    - $\Delta E_{int} = -W = -nC_v\Delta T$
    - Molar Specific Heat of Gas
      - C_v - constant volume
      - C_p - constant pressure
      - $C_p - C_v = R$
      - $\gamma = \frac{C_p}{C_v}$
      - monatomic $C_v = \frac{3}{2}R$
      - diatomic $C_v = \frac{5}{2}R$
    - Adiabatic Expansion
      - For ideal gas, $\Delta E_{int} = nC_v\Delta T$ (only depends on temp)
      - $PV^\gamma = k$
      - $W = \frac{P_fV_f - P_iV_i}{\gamma - 1}$

- 2nd Law of Thermodynamics
  - heat doesn't flow spontaneously from cold to hot
  - Heat Engine - cycle, $\Delta E_{int} = 0$
    - $\underset{T_H}{Q_H} \xrightarrow{W} \underset{T_L}{Q_L}$
    - $e = \frac{W}{Q_H} \Rightarrow$ $Q_H = W + Q_L$, $e = 1 - \frac{Q_L}{Q_H}$
  - Kelvin Planck - no heat engine all work (100%)
  - Carnot Engine
    - Reversible - so slow that can consider sequence of equilibrium states so can move back and forth w/ no change; very slow = quasistatic
    - Irreversible - not slow, all real processes
    - Carnot Cycle - reversible steps



1 - isothermal, T_H, $+Q_H$
2 - adiabatic, $T_H \to T_L$
3 - isothermal, T_L, $-Q_L$
4 - adiabatic, $T_L \to T_H$

    - $e = 1 - \frac{T_L}{T_H}$
  - Otto Cycle (cars)



1 - adiabatic, gas enters
2 - isovolumetric $+Q_H$ (spark)
3 - adiabatic (power stroke)
4 - isovolumetric (exhaust stroke) $-Q_L$

    - $e = 1 - \left(\frac{V_a}{V_b}\right)^{1-\gamma}$

- Carnot Refrigerator (Heat Pump)
  - $\underset{T_L}{Q_L} \xrightarrow[W]{} \underset{T_H}{Q_H}$
  - Coefficient of Performance - $COP = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$ ← for cooling
    - $COP_{ideal} = \frac{T_L}{T_H - T_L}$
  - Heat Pump (heating) $COP = \frac{Q_H}{W}$
    ← efficiency measured w/ heat moved

- Entropy
  - $\Delta S = \frac{Q}{T}$, $dS = \frac{dQ}{T}$, $\Delta S = \int_a^b \frac{dQ}{T} = S_b - S_a$
    ↑ only for reversible
  - state variable.
  - can approximate any path w/ series of Carnot cycles
  - $\oint \frac{dQ}{T} = 0$
  - for irreversible, equal to reversible, approximate w/ average
  - For reversible $\Delta S = 0 = \Delta S_{sys} + \Delta S_{env}$
    - IRR, $\Delta S = \Delta S_{sys} + \Delta S_{env} > 0$

- Extra
  - Fick's Law - $J = DA\frac{dC}{dx} = DA\frac{C_1 - C_2}{\Delta x}$
    ↑ diffusion rate
  - $P_{atm} = 101325$ Pa
  - $P = \rho gh$
  - For V_{rms}, remember it is sum of 3 directions
  - $\frac{3}{2}NkT$ applies to all molecules
  - average velocity of gas is 0 (symmetry)
  - $PM = \rho RT$
  - don't forget to make f(v) a probability distribution
  - ΔE_{int} only depend on temp
  - draw diagrams
    - adiabatic steeper
  - Use $PV = nRT$ to compare temps on PV
  - friction ⇒ irreversible
  - For ΔS, integrate if can
  - $\frac{dQ}{dt} = kA\frac{dT}{dx}$ is useful also for the equilibrium

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Physics 7B
MT2

- opposites attract, like repel
- Law of conservation of electric charge
- charge leakage to polar water molecules
- conductors, semiconductors, insulators
- induced charge, grounding wire
- Coulomb's Law - $F = \frac{kQ_1Q_2}{r^2}$
  - $k = 8.99 \cdot 10^9 \ Nm^2/C^2$
  - $k = \frac{1}{4\pi\varepsilon_0}$, $e = 1.602 \cdot 10^{-19}$ C = quantized
  - $\varepsilon_0 = 8.85 \cdot 10^{-12} \ C^2/Nm^2$
    - Permittivity of free space
- Electric Field - $\vec{E} = \vec{F}/q$, +test charge
  - $E = \frac{kQ}{r^2}$, $\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$; N/C, V/m
  - superposition principle
  - F_{ab} - force on a by b
  - $\vec{E} = \int d\vec{E}$, ρ, σ, λ
  - infinite plane $E = \frac{\sigma}{2\varepsilon_0}$, infinite wire $E = \frac{\lambda}{2\pi\varepsilon_0 R}$
  - lines out of +, into -
    - proportional to # of charge
  - E-field inside conductor = 0
  - conductor - charges distribute evenly outside
    - E-field always ⊥ to surface
- Electric Dipole
  - $+Q \overline{\ \ell\ } -Q$, dipole moment, $p = Q\ell$
  - $\vec{p}$ points from negative to positive
  - $\vec{\tau} = \vec{p} \times \vec{E}$, $\tau = pE\sin\theta$
  - $E = \frac{kp\cos\theta}{r^3}$, $V = \frac{kp\cos\theta}{r^2}$
- Electric Flux
  - $\phi = E \cdot A \cdot \cos\theta$, $\phi = \vec{E} \cdot \vec{A}$
  - $\phi = \int \vec{E} \cdot d\vec{A}$
- Gauss's Law
  - $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$
  - Use lots of symmetry
  - Use negative charge to cancel out a charge
- Electric Potential
  + $\Delta U = -W$; $W = Fd = qEd$
  + $V = \frac{U}{q} = -\frac{W}{q}$ J/C
  - define $V = 0$ at $r = \infty$
  - $V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell}$, $V = E \cdot d$
  - $E = -dV/dx$, + can do partial derivatives $-\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z}$
  - $V = \frac{kQ}{r}$, point charge, $V = 0$ at $r = \infty$
  - $V = k\int \frac{dq}{r}$, any charge distribution
  - equipotential lines are ⊥ to E-field
    - density field lines ∝ E field strength
    - entire volume of conductor is equipotential
  - $1eV = 1.6 \cdot 10^{-19}$ J
  - air breakdown, $E = 3 \cdot 10^6$ V/m
- Capacitors
  - $Q = CV$, Farad (F) = C/V
  - parallel plate - $C = \frac{\varepsilon_0 A}{d}$
  - single conductor - $C = 4\pi\varepsilon_0 r_b$
  - mostly reverse of resistors
  - series - same charge, $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$
  - parallel - same voltage, $C = C_1 + C_2 + \dots$
  - $W = \int V dq$
  - $E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$
  - $u = \frac{1}{2}\varepsilon_0 E^2$, energy density
- Dielectrics
  - permittivity $\varepsilon = K\varepsilon_0$
  - $C = K\varepsilon_0 \frac{A}{d}$, $u = \frac{1}{2}K\varepsilon_0 E^2$

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Final

- Electric Circuits and Resistance
  - $I = \frac{dQ}{dt}$ Ampere(A)
  - conventional current: $+ \to -$
  - Ohm's Law: $V = IR$
  - $R = \rho \frac{\ell}{A}$ resistivity $\rho := [\Omega \cdot m]$
  - $\sigma = \frac{1}{\rho}$ conductivity
  - $\rho_T = \rho_0 (1 + \alpha(T - T_0))$ (small ΔT)
  - $P = IV = \frac{dU}{dt} = \frac{dq}{dt} V$ $(U = qV)$
    - $P = I^2 R = \frac{V^2}{R}$
  - AC: $V = V_0 \sin(2\pi f t) = V_0 \sin(\omega t)$
    - V_0 is peak voltage
    - $I = \frac{V_0}{R} \sin(\omega t) = I_0 \sin(\omega t)$
    - $P = I_0^2 R \sin^2(\omega t)$
    - $\bar{P} = \frac{1}{2} I_0^2 R = \frac{1}{2} \frac{V_0^2}{R}$
    - $V_0 = \sqrt{2} \cdot V_{rms}$; $I_0 = \sqrt{2} \cdot I_{rms}$
  - electric field in wire w/ voltage
    - $\vec{E} \neq 0$ b/c not static
  - Current Density $\vec{j}$; $I = \int \vec{j} \cdot d\vec{A}$
  - Drift Velocity $\vec{v}_d$
    - $I = nAv_d$; n: C/m^3
    - $I = neAv_d$; n: e^-/m^3
    - $\vec{j} = ne\vec{v}_d$
    - ≈ 0.05 mm/s
  - $R = \rho \frac{\ell}{A}$; $I = jA$; $V = E\ell$; $\vec{j} = \sigma \vec{E}$
  - Superconducting: $\sim 0K$, $\sim \Omega \cdot m$
- DC Circuits
  - electromotive force (emf) $\mathcal{E}$
  - battery has internal resistance
    - terminal voltage: no load
    - $V = \mathcal{E} - Ir$
  - resistors
    - series: $R_{eq} = R_1 + R_2 + \ldots$
    - parallel: $R_{eq} = (\frac{1}{R_1} + \frac{1}{R_2} + \ldots)^{-1}$
      - b/c charge conserved
  - Kirchhoff's Rules
    - 1) $I_{in} = I_{out}$
    - 2) $\Sigma V_{loop} = 0$
  - RC Circuit
    - $V_C = (1 - e^{-t/RC}) \cdot V_0$ q
    - $\tau = RC$ time constant, time to 63%
    - $V_C = V_0 e^{-t/RC}$ ✓37%
  - galvanometer - deflection ∝ I, sensitive
  - ammeter - galvanometer ∥ shunt resistor
  - voltmeter - galvanometer series big resistor
  - ohmmeter - battery + galvanometer
  - sensitivity - $\frac{\Omega}{V}$ resistance at certain range
- Magnetism
  - Poles - no monopole, N → S
  - current makes magnetic field
  - Right Hand Rule B, I, conventional, F
  - $F = I\ell B \sin\theta$
    - wire in magnetic field
  - Tesla (T) $1T = 1 \frac{N}{A \cdot m}$
  - $\vec{F} = q\vec{v} \times \vec{B} = qvB\sin\theta$ (good for $F = \frac{mv^2}{r}$)
  - Lorentz Equation: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
  - Magnetic Dipole Moment
    - $\tau = NIAB\sin\theta$ atoms can have this b/c orbits!
    - $\vec{\mu} = NI\vec{A}$
    - $\vec{\tau} = \vec{\mu} \times \vec{B}$
    - $U = -\vec{\mu} \cdot \vec{B}$
  - Galvanometer: $\tau = k\phi$; $\phi = \frac{NIAB\sin\theta}{k}$ radial magnet
  - Hall Effect
    - showed e^- is what flows
    - $\mathcal{E}_H = E_H d = v_d B \cdot d$
- Sources of Magnetic Field
  - long wire: $B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}$ $\mu_0 = 4\pi \cdot 10^{-7} \frac{T \cdot m}{A}$
    - parallel wires: ↑↑ attract, ↑↓ repel
  - Ampere's Law: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{encl}$
  - Solenoid: $B = \mu_0 \cdot \frac{N}{L} I$
  - Biot Savart Law
    - $d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{\ell} \times \hat{r}}{r^2}$
    - $dB = \frac{\mu_0 I \, d\ell \sin\theta}{4\pi r^2}$
    - $\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\ell} \times \hat{r}}{r^2}$
- Induction
  - Magnetic Flux: $\Phi_B = BA\cos\theta = \int \vec{B} \cdot d\vec{A}$
    - $1\,Wb = 1\,T \cdot m^2$
  - Faraday's Law of Induction $\mathcal{E} = -\frac{d\Phi_B}{dt} \cdot N$
    - tries to keep magnetic field already there
    - generate B field opposite, current too
    - Lenz's Law aka
  - Moving conductor $\mathcal{E} = B\ell v$
    - any rod moving thru B field
    - think of it as e^- moving, so have a force
  - Back emf in motor, counter torque in generator
  - eddy current - insulate sheets ↑ resistance, ↓ power loss
  - transformer - $P_{in} = P_{out}$, $\frac{V_s}{V_p} = \frac{N_s}{N_p}$
  - Electric Field by Changing Magnetic Field
    - $\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$
    - nonconservative, forms closed loops
- Extra
  - induced emf for moving rods is just area covered by it per time
  - $A_\theta = \frac{1}{2} \theta r^2$
  - field fringing always exists
    - use Ampere's Law of Faraday's to show
  - Gauss Law: $\oint \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$
  - Dipole E field (Far): $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{x^3}$, $p = d \cdot q$
  - $C = \frac{K\varepsilon_0 A}{d}$
  - $P = Fv$
  - $\gamma = \frac{C_p}{C_V}$